UNIVERSITY OF ECONOMICS - VARNA

FACULTY OF INFORMATICS

DEPARTMENT OF STATISTICS AND APPLIED MATHEMATICS

Julieta Deyanova Mihaylova

OPTIMIZATION OF MATERIAL FLOWS OF FAST TURNOVER GOODS IN A SMALL WHOLESALE AND RETAIL COMPANY (THE EXAMPLE OF MALVIS TRADE LTD)

A B S T R A C T

of dissertation thesis
for the award of the educational and scientific degree "PhD"
in professional field 3.8. Economics,
scientific specialty Optimal Management of Economic Systems

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Figures - 32

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The dissertation thesis was discussed at an extended meeting of the Department of Statistics and Applied Mathematics at the University of Economics - Varna.

The author is a PhD student at the Department of Statistics and Applied Mathematics at the University of Economics - Varna. The research and development were carried out at the same university.

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I. GENERAL CHARACTERISTICS OF THE THESIS

1. Relevance of the topic

In today's dynamic economic reality, businesses face a number of challenges and difficulties. Companies need to adapt effectively to changes in the market environment in order to maintain and improve their positions, secure competitive advantages and meet emerging market needs. To this end, sound decisions must be taken on a daily basis, based on analyses of the internal and external environment.

Analyzing the external environment is a complex process, as the information needed for this is not always easily available. In addition, force majeure circumstances may arise in the external environment which are practically unforeseeable but have a significant impact on the companies' operations. It is therefore essential that the management of the company has a good knowledge of its activities, strengths and weaknesses in order to be able to manage it in the best possible way.

Every enterprise, irrespective of the area in which it operates, needs various production factors in order to be able to carry out its activities. In practice, they are the inputs that participate in the production process and directly influence the firm's activities. The factors of production are land, labour, capital, entrepreneurship, energy and information.

One of the main characteristics of the production factors (resources) is their *scarcity*. This is not characterized by a shortage of a particular resource, but rather by the fact that it must be used rationally to achieve the greatest satisfaction of needs at the lowest possible cost. This involves choosing the best option for allocating and using available resources.

In order to find the optimal option for the allocation of the resources available to the company, the working method must be thoroughly studied and analysed and various options for its optimisation must be explored. In modern economics, various methods are used to study, analyze and manage economic entities. In the current economic environment, serious consideration needs to be given to the various economic and mathematical methods and models that would be useful for optimising the performance of various enterprises.

2. Object and subject of study

The object of research in this dissertation is the delivery of goods and the transportation activities of a small company trading with food products (Malvis Trade Ltd.).

The subject of the study is the optimization of deliveries and transportation routes in order to reduce the cost of goods storage and transport while maximally meeting the needs of the company's customers.

3. Purpose and objectives of the study

The purpose of the dissertation thesis is to develop and test models for optimization of the planning process based on the analysis of the current state and development of the supply and transport activities of Malvis Trade Ltd.

In order to achieve the set purpose, the following main **objectives** should be solved:

1. A theoretical review of the modern methodological basis for optimizing the movement of material flows is to be made, as well as to study and analyze the way of working of the enterprise - the organization of activities on requesting, receiving, storing, selling and transporting goods and material values.
2. The means of economic-mathematical modeling as a means of optimization are to be presented and, on this basis, to propose author's methodologies for improving the above activities.
3. The results and analyses of the application of the author's proposed methodologies in the real activity of Malvis Trade Ltd are to be presented.

4. Research thesis of the dissertation

This dissertation argues that optimization of supply planning and optimal routing can be accomplished using appropriate economic-mathematical models.

5. Research methodology

In the course of the analysis various research methods were used to solve the set tasks and achieve the main objective. SWOT-analysis has been used to present the strengths and weaknesses of the enterprise, through which potential opportunities and threats have been analyzed. Optimization in supply planning was carried out using uncertain demand modelling methods. Elements of graph theory and linear optimization methods were used to optimize the firm's transportation activities. Data analysis was performed using the R programming language and the built-in MS Excel functions.

6. Aprobation

Real opportunities for optimization of supplies of goods and materials and of routes for distribution of sold goods applicable in the activity of Malvis Trade Ltd. are revealed, showing that not all optimization models are applicable in certain cases.

The developed methodologies are also applicable to the activities of other companies operating in the trade and/or transport sector.

Some of the theoretical conclusions and practical applications are presented in research papers and articles.

7. Table of contents

II. BRIEF OUTLINE OF THE DISSERTATION THESIS

Chapter one

THEORETICAL AND METHODOLOGICAL FOUNDATIONS OF THE MOVEMENT OF MATERIAL FLOWS

The first chapter of the dissertation is focused on the study of the theoretical and methodological foundations of the activity of commercial companies. The **first paragraph** discusses the essence and content of logistics and supply chains. Through a critical analysis, the main factors determining their importance for the performance of enterprises are outlined.

In order to analyze the activities related to the operation of trading companies, a critical review is made of various authors' views on the definition of a number of concepts that are bound up with them, namely 'logistics', 'logistics product', 'logistics function', 'logistics chain', 'supply chain', 'supply chain management', as well as the links and dependencies between them.

Supply chain is one of the basic concepts closely related to logistics. It can be seen as the set of all organizations and activities involved in sourcing raw materials, manufacturing and transporting the finished product to the end user. Supply chains can vary in length and complexity depending on the number of participants involved. A direct supply chain has only three participants - a supplier, a company (manufacturer or distributor) and a customer (*Figure 1*).

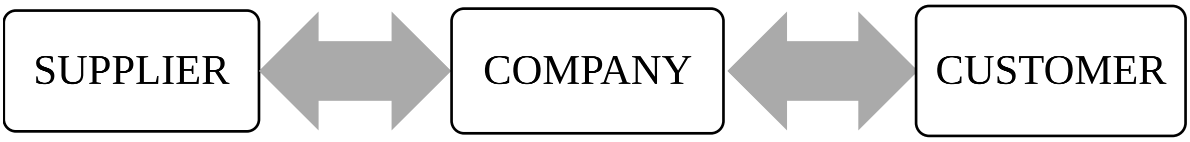

Figure 1 *Direct supply chain*

The extended supply chain includes suppliers who supply the company's main supplier and two types of customers - wholesale and retail (*Figure 2*).

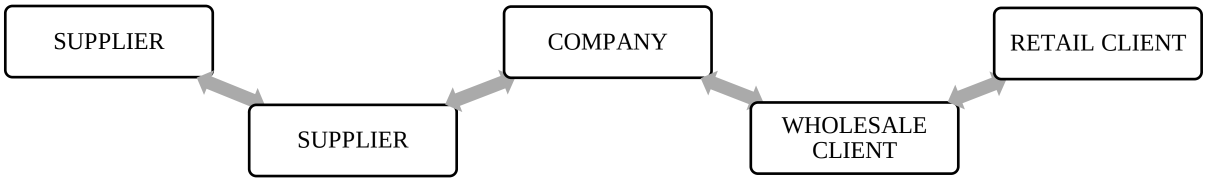

Figure 2 *Extended supply chain*

The maximum supply chain includes all organizations that have any involvement in the movement and processing of raw materials, finished goods, information, etc. (*Figure 3*).

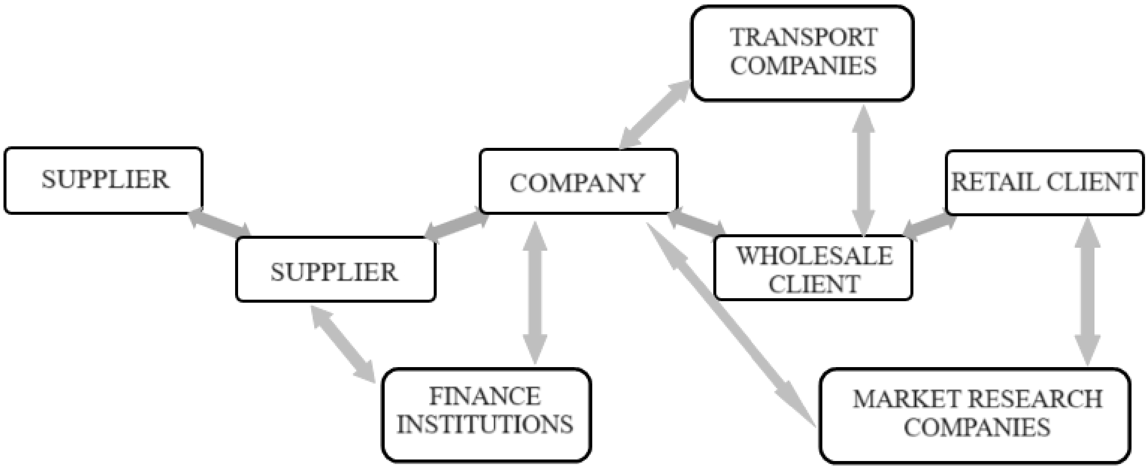

Figure 3 *Maximum supply chain*

Participants in each supply chain must make decisions together and separately in the areas of production, inventory, location of production and storage areas, transport and information. Together, these decisions determine what opportunities exist and how efficient a supply chain is.

Together with the concept of supply chain, the concept of supply chain management is defined. Supply chain management is the coordination of the location, production, inventory and transportation of goods between supply chain participants to achieve the highest efficiency in the market being served. It encompasses the planning and management of all activities involved in sourcing, conversion and all logistics management activities. It also includes coordination and cooperation with partners, which may be suppliers, intermediaries, service providers and customers.

In general, supply chains cover three flows - information, financial and material. The three flows move through the whole chain, in all directions, and all participants are bound by them.

Material flows represent all the goods and material values that at a given point in time make their way through the supply chain. They are formed as a result of the activities of different enterprises that produce and consume resources. Flows are in continuous movement between and within these enterprises. The objective of any organization is to arrange the movement of material flows so as to minimize its costs and optimize its activities. This is done through material flow analysis, which is an assessment of the flows and stocks of materials within a particular system.

In the **second paragraph**, the importance of material flows and transport activities for companies' operations is presented in more detail. Material flow analysis enables the study of any system, with the possibility of combining it with other methods appropriate in particular situations and thus studying and optimizing system performance.

In the course of the company's activities, it is possible that the material flow may temporarily cease its movement. At this point (period) in time, it is considered as a inventory. Inventories are formed in all phases of activity, and their classifications are extremely diverse.

Commercial enterprises play a crucial role in distribution channels, helping to move goods from producers to final consumers. In trade, new products are not produced, but the operations carried out transform material flows from one type to another. In this area, inventories are of particular importance to the activities of companies.

Wholesalers specialize in purchase and sale, but also perform other logistical functions such as transport, storage, sorting, marketing communications, etc. In carrying out their activities, they reduce the number of transactions between manufacturers and retailers.

In order for firms to meet customer needs, they must have a very good knowledge of the market and consumer preferences, and be able to meet demand for goods as requests arise. This can be done in two different ways.

The first way involves purchasing the requested goods once the customer's order has been received. However, this leads to a delay in deliveries.

The second way of meeting demand is by maintaining inventory. This means that the company incurs certain additional costs in storing this inventory, but it can satisfy customer's needs as they arise without incurring delays.

There are different authors' opinions on the reasons for maintaining inventories, but in all cases the thesis that maintaining inventories provides an advantage for companies is proved. Maintaining inventories requires additional costs on the part of companies, but the revenues from properly maintained inventories would exceed the costs of maintaining them.

In order to maintain optimal inventories of goods to cover the company's needs, the following questions must be answered:

1. How often should the inventory of a particular product be tracked?
2. When should a replenishment order be placed?
3. What should the order size be?

To answer these questions the following factors must be taken into account: demand, delivery time of goods, available stocks and tracking them, shelf life of goods and reparability. The degree of influence of these factors is determined by the company's field of activity.

The availability of inventories is determined by the structure and activity of the company. In determining the quantity of inventories, account must be taken of all the characteristics of the commercial enterprise and the environment in which it operates.

A special case are enterprises engaged in trading with groceries, since this type of goods is relatively perishable and maintaining large inventories would lead not only to more costs, but also to serious losses for the company if the production does not sell within the specified shelf life.

Transport logistics is the scientific organization of the management of material, information and financial flows, vehicles and transport-related operations (loading and intermediary activities). It organizes and implements all operations of loading, transporting, unloading, temporary storage etc. of goods using various vehicles and equipment. Transportation can be defined as a key logistics function related to the movement of products by vehicles in the supply chain and consists of logistics operations including dispatch, handling, packaging, transfer of title to the goods, insurance against risks, etc.

The task of transport is to move the materials or production from the producer to the consumer, and this consumer may be a retail customer or a production unit. A special feature of transport is that it does not create a new material product, but continues and supports the production process of the company. However, it is involved in determining the value of the goods and is one of the most important logistical functions.

The choice of method of transport and of transport company are essential for the operation of firms, as inappropriate transport can negatively affect both the production process and customer satisfaction.

The preparation of optimal transportation plans is also linked to the preparation of freight transport routes. They should be defined in such a way as to ensure the rapid movement of goods between traders and customers and the lowest possible transport costs. To this end, it is necessary to know the locations of the various sites and to have a good knowledge of the roads linking them.

Transport process management is virtually a system for managing technological and information flows. Information flows accompany the goods, vehicles and resources that are required for their operation.

The planning, implementation and control of transport activities in a company are essential for the optimal performance of its activities. In order to make the right decisions, a number of factors must be taken into account, including the nature of the activity being carried out, the capabilities of the company, the type of goods being transported and the needs of customers.

Inventory management and the transport activities of commercial companies are essential for the optimal performance of their activities.

The **third paragraph** presents the object of the research – the company Malvis Trade Ltd. and its activities.

The company was founded in 1991, as general partnership Malvis-Boyanovi S-IE, and is currently Malvis Trade Ltd. Initially, the company was engaged in retail trade in its own grocery stores in the city of Varna. In 1998, the first wholesale warehouse of the company started to operate, from which the company replenished its stores in the city of Varna and other settlements in the Varna region. Over the years, the location and number of the company's warehouses have changed, and currently the activity is carried out with its own warehouse area and its own vehicles.

The research interest in this dissertation focuses on the optimization of the supply of goods and the routes used to distribute the sold goods. The structure of the company and the organization of the activities are described, which allows for a better understanding of the subject of study.

Malvis Trade Ltd owns a wholesale warehouse and its own vehicles, which are used for the distribution of goods. The vehicles are eight in total of two types and have different load capacities. The company uses warehouse software that allows for constant tracking of the available quantities of goods and their movement.

The large assortment of goods on offer (over 1100 items) and the large number of customers (over 1000) makes business planning extremely complex. In addition, the different types of products have different shelf lives (perishable, medium shelf life and long shelf life), which means that replenishment takes place over different periods of time and storage of the goods is for different periods. Currently, supply planning is done at the discretion of management based on available quantities.

Each vehicle travels to several customers on each run when delivering the goods sold. It is of utmost importance that the runs are planned and distributed in the best possible manner. Currently this is done at the discretion of the warehouse workers and drivers.

SWOT analysis reveals the strengths and weaknesses of the company, as well as the opportunities and threats facing it. This enables the systematization of the main development opportunities.

Strengths:

- work in own warehouse and office, which means no extra rental costs;
- all activities take place in one building, which facilitates communication and document flow;
- storage costs are virtually independent of inventory levels;
- the use of own vehicles, which facilitates the planning of transport activities;
- the availability of an online shop through which the company's customers place orders, thus facilitating the work of the staff;
- a photovoltaic system, the use of which significantly reduces the company's electricity costs.

Weaknesses:

- relying entirely on foreign transport for the supply of goods, which makes the goods on offer more expensive and can lead to delays or non-delivery;
- the use of obsolete vehicles, leading to more frequent breakdowns and higher fuel costs;

- not using an integrated business management system (ERP, CRM, BI) but relying on separate software products for each activity, which slows down and complicates staff work.

Opportunities:

- expanding the business by finding new suppliers and thus increasing the range of products offered;
- attracting new customers, which would expand the markets in which the company operates.

Threats:

- high levels of competition in the sector, which can lead to customer churn due to low satisfaction or due to more favorable terms offered by competitors.

The tracking and planning of the movement of goods and the planning of transport activities are essential for the optimal performance of the activities of Malvis Trade Ltd. In order to achieve maximum and accurate result in the research, actual data of the company's operations for the period 2017-2023 provided by its management has been used.

Chapter two

SUPPLY MODELLING UNDER UNCERTAIN DEMAND

The research in chapter two is focused on the analysis of the possibilities of modelling the supply of goods and services under uncertain demand. An overview of some of the main issues involved is given. It is shown that consumption of goods and commodities can be represented by time series and an overview is given of the methods and models that can be used to analyze and model them.

A methodology is developed to model the supply of goods under uncertain consumer demand, with the objective of satisfying customer needs as fully as possible. Consumer demand depends on many factors and is not constant. Changes in consumption over time may be due to changes in consumer preferences, changes in the quality or prices of goods, or momentary fluctuations due to force majeure (e.g. the COVID-19 pandemic).

In developing the methodology, the shelf life of inventory is also essential. How the consumption analysis will be done depends on how long it is:

- durable products - with a shelf life of a few months to 2 years. The consumption analysis is based on monthly consumption. It is possible to separate a trend, a seasonal component and a random component. The data to be taken into account in modelling consumption should cover several (at least four) annual periods;
- medium shelf life products - with a shelf life of two to three weeks. Consumption is analyzed on a weekly basis. In this case, a trend and a random component can be separated. The large fluctuations in consumption during the weeks, which have the same number in the year, do not allow the seasonal component to be separated. In analyzing the consumption of these products, data relating to time periods remote from the current moment have negligible impact on current consumption and can be disregarded;
- perishable products - with a shelf life of three to seven days. Consumption is analyzed day by day. Data for periods close to the current time may be used in the analysis.

A methodology has been developed for planning on the basis of weekly consumption with the following notations: q is the quantity of the given product, $\xi = q$ is the random variable of its consumption, and d is the demand. Let, moreover, $100p_1$ is the percentage of satisfied customers at first delivery, and $100p_2$ – the percentage of satisfied customers at second delivery. The distribution function of the random variable ξ is $F_{\xi}(q)$, and the level of the transitional inventory is q_0. Then the quantity of the first delivery will be:

$$q_1 = \max\left\{\left[F_{\xi}^{-1}(p_1) - q_0\right], 0\right\}. \quad (1)$$

The meaning of (1) is to account for the existence of a transient inventory. If the transient inventory is less than the required stock, then a supply is scheduled equal to the difference between the two stocks. If the transient inventory exceeds the requirement, the difference is negative and then no delivery is scheduled (or the scheduled delivery is zero).

The residue (surplus) is:

$$q_r = \max(q_1 + q_0 - d, 0). \quad (2)$$

The shortage is:

$$q_d = \max(d - q_1 - q_0, 0). \quad (3)$$

If the shortage is greater than zero, i.e. $sign(q_d) = 1$, a second delivery is planned in the amount of:

$$q_2 = \left[F_\xi^{-1}(p_2) - F_\xi^{-1}(p_1)\right] \cdot sign(q_d). \quad (4)$$

The final amount of the surplus (or shortage) shall be calculated using the following formulas:

$$q_r = \max(q_0 + q_1 + q_2 - d, 0). \quad (5)$$

$$q_d = \max(d - q_0 - q_1 - q_2, 0). \quad (6)$$

The block diagram of the algorithm described above is presented in *Figure 4*.

Monthly supply planning allows for the seasonal component of consumption to be accounted for. The author's suggestion is to build the models both on a seasonal basis (four seasons per year) and on a monthly consumption basis, choosing a model that is adequate.

In the case of perishable products, supply planning must be carried out with particular care. It is necessary to schedule deliveries twice during the working week, and the author's proposal is to artificially divide the weekly period into two three-day periods (excluding the rest day Sunday) and to make deliveries on Mondays and Thursdays. In contrast to weekly planning, in semi-weekly planning no additional delivery is technologically possible to meet increased demand during the corresponding three-day period. It is appropriate to collect advance orders from retailers and to plan additional minimum quantities to meet occasional needs. Advance orders can be considered as a deterministic component of the time series of demand for a given product.

The search process for each type of product is highly specific and different time series have different theoretical models (not only a difference in model parameters, but also a difference in the models in essence). Hence, the task of discovering the best model describing the demand for a given product can be posed. When comparing two models, the one that is closer to the observed values is considered to be the better one, the estimate of this approximation being equal to the sum of the squares of the deviations. Thus, the best model will be the one with the lowest estimate.

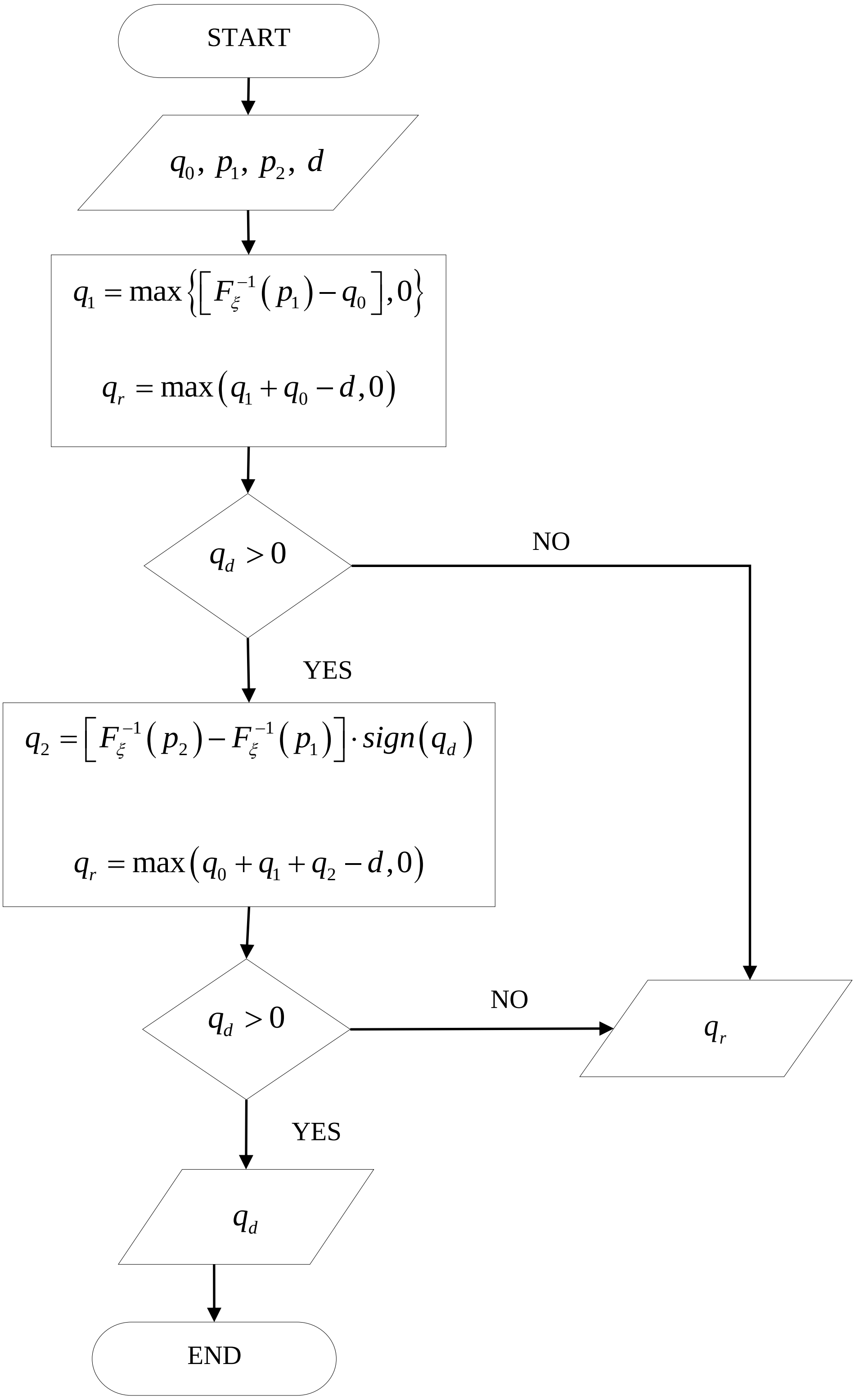

Figure 4 *Supply planning block diagram for replenishment*

Therefore, several theoretical models have to be created for each time series. Holt's models and the AR, MA, ARMA, ARIMA models developed by Box and Jenkins are well known. According to the author, it is appropriate to choose a demand model for each individual product from among them, and a methodology has been developed to evaluate the models.

Let the values y_1, y_2, …, y_t, …, y_n of the time series are known. For each member of the time series y_t theoretical models are created:

$$\varphi_s\left(t, Y_{t-1}, \mathrm{E}_{t-1}\right), \text{ s=\{Holt, AR, MA, ARMA, ARIMA …\}}, \qquad (7)$$

where:

t is time;

$Y_{t-1} = \left(y_1, y_2, ..., y_{t-1}\right)$ is a vector with the values of the previous members of the time series;

$\mathrm{E}_{t-1} = \left(\varepsilon_1, \varepsilon_2, ...\varepsilon_{t-1}\right)$ is the vector of deviations of the model from the actual values for the previous members of the time series.

With the models created, predicted values are determined for each member of the series:

$$\hat{y}_t = \varphi_s\left(t, Y_{t-1}, \mathrm{E}_{t-1}\right). \qquad (8)$$

The deviation of the estimated value from the actual value is determined:

$$\varepsilon_t = y - \hat{y}. \qquad (9)$$

The sum of the squares of all deviations of the observations from the predicted values up to the member with number t gives a non-normed estimate of the accuracy of the model. When comparing the estimates of different models on the same time series, it is appropriate to select the model with the lowest estimate. The prediction of $n+1$-st member of the series is made with the selected model. It is considered to be closest to the expected actual consumption.

Sales data from 2017 to 2023 were used in the consumer demand analysis. The analysis is hampered by the large number of items and the fact that the consumption of most of them is not observed throughout the study period. In some cases, the consumption of one product can be substituted by a similar product produced by the same or another manufacturer. Such substitution occurs, for example, in the case of yoghurts. In such cases, the analysis of consumption can be made not for each individual product but for a whole group of similar products.

An attempt is made to create a model of yoghurt consumption. As input data, consumption in 2020-2022 is considered. On the basis of the model proposed by the author, consumption for 2023 is forecasted and the forecasts are compared with actual consumption. Several methods are used with the aim of finding the most appropriate one for forecasting. The dummy variable method, Holt's one-parameter model (exponential smoothing), Holt's two-parameter model and ARIMA models are used. The normality of the distribution is verified with the Shapiro-Wilk test and the trend and variance were verified with the Foster-Stuart test. The following conclusions are drawn from the tests conducted and results obtained:

- when using the dummy variable method, there is a significant difference in the predicted versus actual data. It follows that it is not adequate;
- forecasting with one-parameter Holt models is adequate on real data;
- according to the tests performed and the results obtained, forecasting with a two-parameter Holt model is acceptable;
- the results obtained show that forecasting with ARIMA models is adequate.

Adequate models can be compared in terms of what the residual variance is. The models are compared two by two by calculating the statistics.

$$F = \frac{s_1^2}{s_2^2}, \quad (10)$$

where s_1^2 is the larger of the two dispersions, and s_2^2 - the smaller. The statistics have a Fisher-Snedecor distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom, as n_1 and n_2 are the volumes of the two samples. The calculated F-statistics for the model pairs are shown in *Table 1*. It can be seen that the prediction with the three models is of the same quality.

	Holt's one-parameter model	Holt's two-parameter model	ARIMA models
Holt's one-parameter model	1	1,003	1,016
Holt's two-parameter model		1	1,018
ARIMA models			1

***Table 1** Estimated F-statistics for the pairs of models used for short-term forecasting of yoghurt consumption*

Consumption forecasting is the basis for applying the author's proposed supply planning and replenishment algorithm. As an example, the distribution of yogurt for the year 2023 is considered. The main objective in this planning is to maximize customer satisfaction. Holt's one-parameter model is used for forecasting. From the results presented, it can be seen that using the above algorithm with the selected parameters, full satisfaction of demand has been achieved with three additional deliveries required. 85.8% of the yoghurt was sold immediately after delivery. 14.2% was sold within 10 days after delivery. There were no rejects.

There is no reason to claim that the application of such an algorithm always leads to such favorable results. Similarly planning models have been created for ayran and fresh milk. The difference is that real historical data for the period 2017 to 2022 were used for modelling. The application of the algorithm is for 2023. The results show that consumption time series are not always ideal. The stationarity of the deviations from the forecast in some cases cannot be accepted unconditionally. In such cases, it is likely that a part of the customers demand is not met.

When forecasting the consumption of durable products, due to their long shelf life, the time series is constructed on a monthly basis. The consumption data of canned fish is used for the study and it can be concluded that consumption is seasonal. Modeling attempts have been made with the dummy variable method and method based on seasonality. In both cases, there is a difference in the forecast and this leads to the conclusion that the models are not adequate.

As with short-term forecasting, Holt models and ARIMA models can be used. In this case, however, restocking is done once during the month. If part of the supply is not sold in the current month it is carried over to the next month. If the demand exceeds the inventory part of the needs will be unsatisfied.

The developed inventory replenishment model was used to plan the supply of durable goods. Consumption data for canned fish, oil and fruit jams were used. The results obtained show that a strict application of the proposed replenishment algorithm may result in some of the needs not being met. Therefore, if there is a risk of running out of stock, provision should be made for additional supplies within the month.

It can be concluded that the proposed algorithm can be successfully applied for supply optimization and for minimization (or complete elimination) of shortages and surpluses of inventory. This, in turn, leads to a reduction in the cost of storing excess quantities (inventory) as well as losses from unfulfilled customer orders.

C h a p t e r   t h r e e

MODELLING AND PLANNING OF TRANSPORT ROUTES

The research in chapter three is aimed at analyzing the possibilities of modelling the transport routes used by the firm for the delivery of goods to its customers. The methods of linear optimization with the help of which the problem can be solved are considered. Particular attention is paid to the Gomory method and the simplex method. The importance of elements of graph theory for the generating of possible routes is shown.

Transportation activities create costs that make the final product more expensive, which necessitates their optimization. The development of a model for the delivery of commodities requires a focus on the main aspects of the activity. It is necessary to define optimality criteria and the constraints that exist objectively or are imposed by the nature of the task at hand.

The main objective of the research model is to create an optimal plan for the movement of goods flows from the company's warehouse to customer sites. The necessary information on the volume of quantities delivered to each customer is obtained by summarizing the orders made to be fulfilled within a predetermined period (working day). Another important information is the transport capabilities of the company. These are related to the technical characteristics of the vehicles used - lifting capacity, operating costs (fuel, depreciation, general consumables). Transport is carried out on existing roads, which requires a well-maintained road network and sufficient data on it.

For the purpose of the thesis the following concepts are introduced:

- transport network - a set of traffic junctions and their connecting elements of the existing transport arteries. The natural abstract model of a transport network is the graph;
- objects - the company's warehouse and customers' sites. The sites are located close to traffic junctions. Within a bounded space (a small town or a neighborhood in a larger city) there may be several customer sites that are grouped together and considered as an aggregated site. We will refer to this aggregated site as *area*. Each area is associated to one traffic junction;
- delivery route - a closed loop connecting several traffic junctions with the start and end point at the company's warehouse. The route may contain

repeated elements of the road network. The main characteristic of a route is its length. An example road network with delivery routes is shown in *Figure 5*;

- course - a route traversed by a single vehicle. When executing the route, the vehicle delivers goods to one or several regions;
- delivery plan - the distribution of deliveries by route. The main objective of the plan is the efficient use of available resources.

The execution of each delivery plan constitutes the execution of courses. Planned vehicles travel a certain distance. The cost of the delivery depends on the vehicle used (fuel consumption, depreciation) and the kilometers travelled.

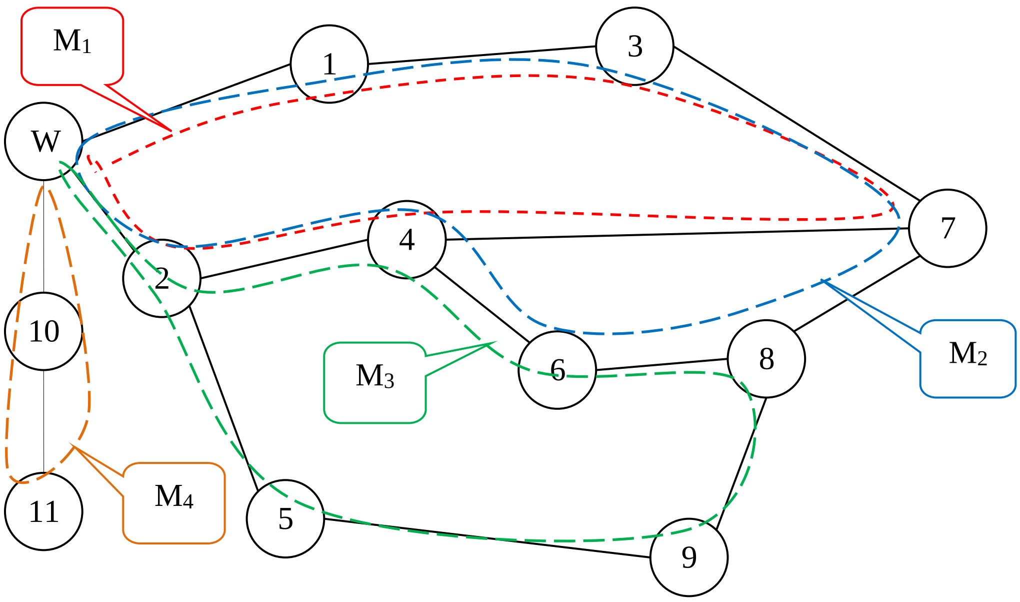

***Figure 5** Transport network*

Therefore, an objective function can be used as a criterion for the optimality of the plan, where the minimum cost of delivering the goods to the company's customers is sought. For this purpose, it is necessary to estimate the cost per unit distance for different types of vehicles. It is convenient to define a given type of vehicle as a reference vehicle, i.e. its operating costs are assumed to be equal to one. For the other vehicle types, corrective efficiency factors are defined which indicate how much of the cost of the reference vehicle is required for each of them.

Let the number of areas to which the commodities are to be delivered be n and the types of vehicles available is k, each type having a different carrying capacity and different operating costs. Let, in addition, all possible routes be defined and the number of them be m. To represent the model, the author introduces the following notations:

- v_s, $s=1...k$ – carrying capacity of the type s vehicle. The gross weight of the load shall be used as the measure of the lifting capacity. Experience has shown that, for foodstuffs, restrictions on the weight of the load outstrip restrictions on the volume of the transport vehicle;
- b_j, $j=1...n$ – the quantity of goods to be delivered to the area j. The measure shall be taken to be the weight of the goods;
- u_s, $s=1...k$ – cost per kilometer of the vehicle type s. The cost per vehicle is a corrective efficiency factor.

Depending on the volume of deliveries on the same route, it is possible to make several courses of the same type or of different types of vehicles. Assuming that the courses are made by the same type of vehicle, the maximum number of courses on a single route may not exceed:

$$d_{ps} = \left\lceil \frac{\sum_{j \in M_p} b_j}{v_s} \right\rceil, \tag{11}$$

where M_p, $p=1...m$ - the multiplicity of the areas associated to the traffic junctions of course p. The symbol $\lceil \ \rceil$ means rounding up.

Uniform course numbering is introduced $i \in \{1,2,...\}$, as the courses with numbers from 1 to d_{11} are operated by the first type of vehicle on the first route, courses with numbers from $d_{11}+1$ to $d_{11}+d_{12}$ are operated by the second type of vehicle on the first route and so on. The total number of courses is $\sum_{p=1}^{m}\sum_{s=1}^{k} d_{ps} = N$. Associated with each course are the performance characteristics of the vehicle running it.

With $x_{q,j}$, $q=1...N$, $j=1...n$ means the quantity of goods to be delivered to area j by course q. As not all routes pass through every area, there are indices (q, j), which are not used.

The following groups of constraints are introduced:

I. $\sum_{j \in M_p} x_{p,j} \le v_s$, (12)

where:

M_p is the multitude of areas served by course p.

v_s is the carrying capacity of the vehicle performing course p.

The constraint expresses the inability to exceed the capacity of the vehicle.

II. $\sum_{p=1}^{N} x_{p,j} = b_j$. (13)

The constraint expresses the need to fully satisfy the needs of the customers of the area concerned.

If the left side of any constraint (13) is zero, the course is not executed. It is necessary to introduce additional binary variables to express the fulfilment or non-fulfilment of the course (if the course is fulfilled, the variable is 1 and if it is not fulfilled, it is 0). We denote them by y_p, $p = 1...N$ and the constraint that applies to them is:

III. $\frac{1}{v_s} \sum_{j \in M_p} x_{p,j} \le y_p \le 1,\ p = 1...N$, (14)

where:

M_p is the multitude of areas served by course p.

v_s is the carrying capacity of the vehicle performing course p.

The nature of the problem imposes a constraint concerning the non-negativity of the variables:

IV. $x_{p,j} \ge 0,\ p = 1...N,\ j = 1...n$, (15)

$y_p \ge 0,\ p = 1...N$. (16)

Let the estimate of the cost of transport on the corresponding course have the form:

$$c_p = S_p u_s , \quad (17)$$

where:

S_p is the length of the route along which the course p is performed;

u_s is the multiplier factor for the operating costs of the vehicle performing course p.

Then the objective function will be:

$$\min : Z = \sum_{p=1}^{N} c_p y_p \ . \qquad (18)$$

In this mathematical model, from a formal point of view, there are always alternative extremums, since the objective function has the same value when renumbering courses within the same route and the same vehicle. From an economic point of view, however, these extremums should not be considered as alternative.

In matrix form, the problem can be written as follows:

$$Z = C \cdot X \ ; \qquad (19)$$

$$A_1 \cdot X \le B \ ; \qquad (20)$$

$$A_2 \cdot X = V \ ; \qquad (21)$$

$$A_3 \cdot X \le \mathrm{O} \ ; \qquad (22)$$

$$X \ge \mathrm{O} \ ; \qquad (23)$$

$$Y \le \|1\| \ , \qquad (24)$$

The elements of the vector V are the values of the lifting capacity of the vehicle performing the course. The coefficients $a_{ij}^{(w)}$ of matrices A_1, A_2, A_3 can take the value 0 or 1. All other elements of the matrices are equal to zero.

The information required to solve the optimization problem is represented in graph form. Its vertices are assigned numerical values (weights) corresponding to the quantity of goods to be delivered to the corresponding area. The edges of the graph are associated with numbers that correspond to the transport costs of the reference transport vehicle

The general structure of the input data may have the form shown in *Figure 6*. With V_i the vertices of the graph are marked. V_1 is the vertice corresponding to the area where the company's warehouse is located. Weights b_i on the vertices are the quantities of inventory to be delivered to the respective customer. The edges are represented by unordered pairs $V_i - V_j$. Weights $W_{i,j}$ of the edges are the costs of travelling the distances between the peaks. From the list of edges, the edge list can be constructed.

Vertices	*Weights*
V_1	b_1
V_2	b_2
V_3	b_3
...	...
V_n	b_n

A. *Graph vertices and their weights*

Edges	*Weights*
$V_1 - V_{i_1}$	W_{1,i_1}
$V_1 - V_{i_2}$	W_{1,i_2}
...	...
$V_2 - V_{i_q}$	W_{2,i_q}
$V_2 - V_{j_1}$	W_{2,j_1}
...	...
$V_3 - V_{j_p}$	W_{3,j_p}
...	...
$V_n - V_{k_1}$	W_{n,k_1}
...	...
$V_n - V_{k_r}$	W_{n,k_r}

B. *Graph edges and their weights*

Figure 6 *Input data structure for the optimization of the delivery of goods*

Determine the closed loops in the graph. Each loop passes through several vertices. The union of the vertices belonging to all contours must be equal to the set of vertices of the graph. Moreover, it is possible that the same vertex participates in more than one loop.

The sum of the weights of all edges belonging to a given loop gives the cost of traversing them using a reference transport. The sum of the weights of all vertices belonging to a loop gives the total volume of goods to be transported along that loop. The following must be taken into account:

- if the same edge is traversed several times during the loop traversal, then its weight contributes several times to the total estimate of the transport cost of traversing the loop;
- regardless of the number of times the same vertex is crossed in the loop, its weight is included only once in the total estimate of goods delivered.

To solve the optimization problem, data on actual sales made by Malvis Trade Ltd. on 01.12.2023 were used. The transport network was created on a digital geographic map (Google My Maps), with individual elements located in separate map layers and depicted with specific symbols.

The creation of the graphic map goes through several stages:

1. Entering the customers sites and the company's warehouse. They are placed in their actual geographical coordinates. The actual names of the sites are used as names. The geographic coordinates are automatically associated to the created objects.
2. Entering the traffic junctions. They are entered as point objects at appropriate locations on country roads and city streets. Traffic junctions' names shall consist of the letter "*N*" and a sequential number. The numbering is arbitrary and is intended to create a unique name for the road junction.
3. Forming of delivery areas. Based on the developer's expert evaluation, some of the traffic junctions are associated with commercial establishments that are in close proximity to them or those to which the junctions provide easy access. Each commercial site may be associated with only one traffic junction, but several different sites may be associated with a single junction. Area names shall consist of the letter “*R*” and the number of the traffic junction to which they are associated. Some junctions may not have associated areas.
4. Forming road sections. Traffic junctions shall be associated with road segments that follow the existing transport network. The length in meters of each section is associated with it. The names of the road segments include the names of the traffic junctions that are ends of the segment. The road section connecting the *Ni* and *Nj* junctions is named *Ni-Nj*.

Given the above, the road network can be modeled with an undirected graph. (*Figure 7*).

For the specific situation on 01.12.2023 the parameters of the problem to be solved are:

- number of supplied sites - 152;
- number of traffic junctions - 59;
- number of areas associated with traffic junctions - 34;
- number of road sections - 91;
- total length of road sections - 234.33 kilometers;
- total gross weight of goods transported - 4399 kilograms.

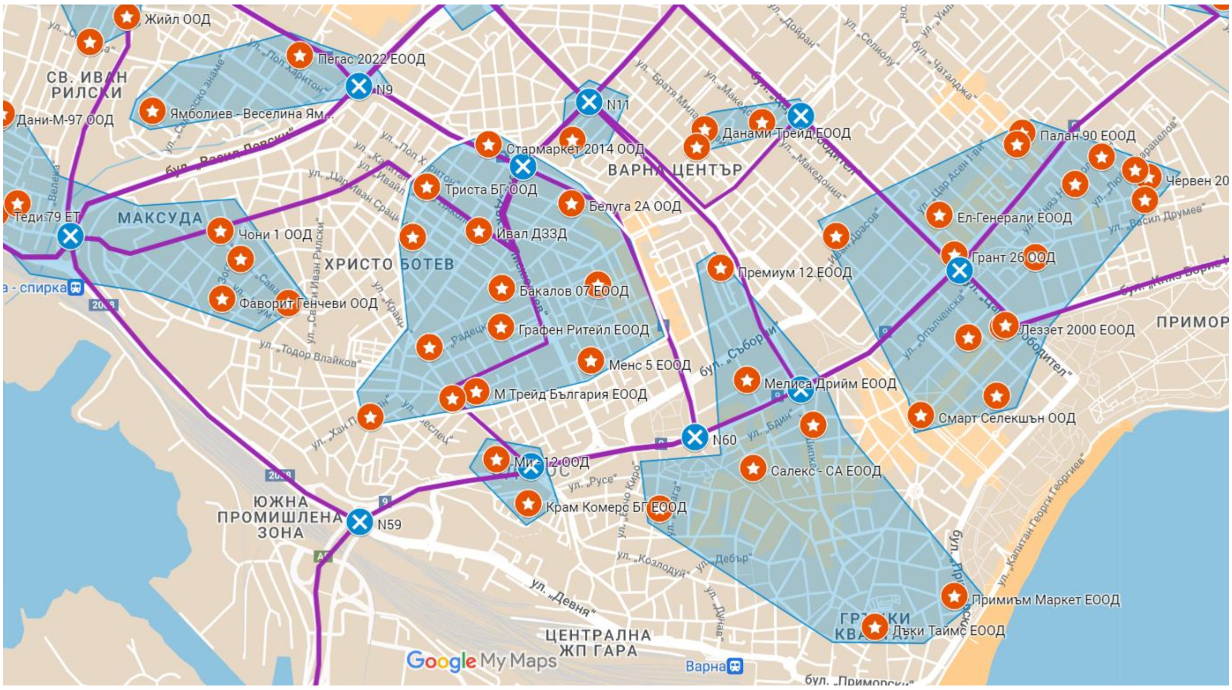

Figure 7 *Malvis Trade Ltd commercial sites, traffic junctions, areas and road sections (part of the geographical map)*

To solve the optimization problem, it is necessary to determine possible supply routes for the areas, and each route must start and end in area 1 (R1, the area in which the company's warehouse is located). The set of routes must be such that it covers all the junctions for which supplies are available. To determine an optimal solution, it is necessary to generate all possible such routes and select those for which the cost of delivering the goods to the customers is lowest. The costs are determined on the basis of the kilometers travelled by each means of transport multiplied by the corresponding efficiency factor.

The R programming language, which provides a free software environment for statistical computation and graphics, was used to create and solve the optimization problem. To create a complete set of closed routes, it is convenient to use the following sequence

1. Determine the shortest path between junction *N1* and an arbitrary junction in the network *Ni* that is associated with a supply area.
2. Determine the shortest path from junction *Ni* to junction *Nj* that is also associated with a supply area.
3. Determine the shortest path from junction *Nj* to junction *N1*.

4. The paths obtained above are sequentially connected and determine the requested route.

For the actions described above, known algorithms such as Dijkstra's can be used. The same has been implemented as functions or packages in various programming languages, including R.

The number of routes created is extremely large. For the example under consideration, the number is 423. The quantities of goods that are delivered to the areas along each route are modelled with variables. The number of these variables for the present example is 10237. The performance or non-performance of a course on a given route is modelled with binary variables. Their number is 1234. The constraints of type I (12) are 1234 in total. Type II constraints (13) are 34 in total. And the constraints of type III (14) are 1234. With the existing mass technical tools, the time to solve such a problem is measured in days and is impractical for daily supply planning.

Due to the enormous size of the problem generated by the above method and the lack of real-time application capability, routes are generated based on expert evaluation. For this purpose, road segments are selected from the transport network that form radial routes from the company's warehouse to the most remote delivery areas and rocade roads that connect selected areas from adjacent radial roads. In this case, it is possible that two or more radial roads have common sections. A set of two radial and one rocade road form a loop. Thus, some of the areas lying on a loop of long length also lie on a loop of shorter length. This allows that if the quantity of goods required to be delivered on a given loop is such that two or more vehicles need to be dispatched, then only one of them will pass on the longest loop.

For the existing road network, radial and rocade roads have been identified in the directions from the Malvis Trade Ltd warehouse to the:

- resort complexes (St. Constantine and Elena, Golden Sands, Vinitsa);
- Chatalja Market - Unification Square;
- central part of the town of Varna;
- neighborhood Hristo Botev;
- South region - south of Varna Lake;
- North-West region - the northern shore of Varna Lake;
- Circumferential route - passes from the South region to the North-West region via the ferryboat in the town of Beloslav.

The defined road sections form the contours presented in *Table 2*.

With the directions and contours thus defined, the linear optimization problem has the following characteristics:

- number of closed routes - 14;
- number of variables modelling the quantities of delivered goods - 316;
- number of binary variables modelling the performance or non-performance of routes - 39;
- Type I constraints (12) - 39;
- Type II constraints (13) - 34;
- Type III constraints (14) - 39.

It is planned to use two types of vehicles - with a capacity of 0.8 tones and 1.5 tones. For the assessment of operating costs, a 0.8 tons vehicle is assumed as the base vehicle (coefficient 1). The operating cost coefficient for a 1,5 tons vehicle is 1,1.

Route	Contour	Length of the contour (km)	Quantity of delivered goods (kg)
Resort complexes	N1-N2-N3-N16-N12-N10-N12-N13-N22-N14-N5-N4-N2-N1	24,4	1229
	N1-N2-N3-N16-N12-N13-N22-N14-N5-N4-N2-N1	14,3	1134
	N1-N2-N3-N16-N22-N14-N5-N4-N2-N1	11,1	1090
Chatalja Market - Unification Square	N1-N2-N4-N5-N14-N22-N7-N11-N6-N24-N1	8,2	747
	N1-N2-N4-N5-N14-N11-N6-N24-N1	6,6	324
Central part	N1-N24-N6-N11-N7-N60-N15-N8-N9-N21-N24-N1	8,9	885
	N1-N24-N6-N11-N8-N9-N21-N24-N1	5,9	733
neighborhood Hristo Botev	N1-N24-N21-N9-N8-N15-N59-N17-N21-N24-N1	8,9	1111
	N1-N24-N21-N9-N17-N21-N24-N1	6,2	715
South region	N1-N24-N21-N17-N59-N50-N53-N55-N56-N57-N45-N44-N45-N57-N56-N55-N53-N50-N59-N17-N21-N24-N1	129,4	1219
	N1-N24-N21-N17-N59-N50-N53-N55-N56-N57-N45-N44-N45-N57-N51-N52-N46-N47-N49-N48-N49-N50-N59-N17-N21-N24-N1	162,9	1510
North-West region	N1-N2-N26-N18-N34-N28-N31-N37-N38-N39-N40-N39-N38-N37-N19-N32-N30-N29-N27-N25-N24-N1	28,4	1248
	N1-N2-N26-N18-N34-N28-N31-N37-N38-N39-N40-N42-N33-N41-N23-N21-N24-N1	30,3	1419
Circumferential route	N1-N24-N21-N17-N59-N50-N49-N48-N47-N46-N52-N51-N52-N46-N43-N42-N33-N41-N23-N21-N24-N1	82,7	1257

Table 2: *Supply routes and loops*

After solving the problem, the following solution is obtained (*Table 3*):

The value of the objective function is 269986.6. It is formed by the length of kilometers travelled by each vehicle in the plan multiplied by the corresponding efficiency factor and is dimensionless.

Vehicle lifting capacity	**Length (km)**	**Contour**	**Supplies area (№-kg)**	**Quantity of delivered goods (kg)**
1,5	24,4	N1 N2 N3 N16 N12 N10 N12 N13 N22 N14 N5 N4 N2 N1	R1-144; R3-41; R16-499; R10-95; R13-44; R22-298; R14-66; R5-8; R4-34	1229
0,8	8,8	N1 N24 N6 N11 N7 N60 N15 N8 N9 N21 N24 N1	R6-51; R11-21; R7-125; R15-27; R8-369; R9-32	625
1,5	162,9	N1 N24 N21 N17 N59 N50 N53 N55 N56 N57 N45 N44 N45 N57 N51 N52 N46 N47 N49 N48 N49 N50 N59 N17 N21 N24 N1	R21-116; R17-423; R50-208; R55-49; R56-17; R57-114; R45-42; R44-106; R51-10; R46-206; R47-40; R48-35	1366
1,5	30,3	N1 N2 N26 N18 N34 N28 N31 N37 N38 N39 N40 N42 N33 N41 N23 N21 N24 N1	R26-57; R34-62; R28-163; R31-145; R39-396; R40-281; R42-75	1179

Table 3: *Procurement plan for 01.12.2023.*

It is necessary to keep in mind that this solution is optimal for the chosen set of loops. As stated above, it was determined by expert evaluation. Choosing a different set would lead to a different possible solution. In this sense, the solution cannot be considered absolutely optimal. An absolutely optimal solution can be obtained if all possible contours are covered. However, this leads to an increase in the number of variables and the number of constraints.

Currently, the transport of goods to the customers of Malvis Trade Ltd. is carried out by two courses by each of the vehicles per day, i.e. a total of 16 courses are made daily. Applying the proposed model offers the option of reducing the number of courses per day and the transport costs. It is important to note that in order to implement this method of operation, it is necessary to change the way in which customers are served, namely to collect all orders for goods during the current day and to deliver them during the next working day.

III. REFERENCE FOR THE CONTRIBUTIONS OF THE THESIS

The theoretical research and the practical studies based on them allow to summarize the following more important contributions in theoretical and applied aspects:

1. On the basis of in-depth research and analysis, the nature of the supply chain, material flows and their management are complemented and clarified.
2. The possibilities of economic-mathematical models and methods for the optimal management of material flows in the supply chain are presented.
3. Economic-mathematical models are developed to help in the optimization of the planning of the supply of goods and in the design of the used distribution routes.
4. Appropriate methods and algorithms are proposed to find optimal solutions of the proposed economic-mathematical models.
5. The applied aspects of the optimizations made are presented through an analysis of the activities of Malvis Trade Ltd.

IV. PUBLICATIONS ON THE DISSERTATION THESIS

Articles:

1. „An Approach to Modeling the Probable Consumers Demand of Food Products Using Pearson Distribution System and Johnson Distribution System.“, Izvestiya : Journal of Economics, Management and Informatics, Varna : Publ. house Science and Economy, 67, 2023, 3, 213-223., ISSN (online) 2367-6957 / DOI 10.56065/IJUEV2023.67.3.213 / https://journal.ue-varna.bg/uploads/20231024113258_6184927586537ab6aa4c10.pdf, individual.

Reports:

1. „Using the Box - Jenkins method to assess the dynamics of wholesales in a small distribution company.“, The role of fundamental programs in university education: Proceedings from international scientific conference, organized by the department of Statistics and applied mathematics of the University of Economics – Varna, 21 oct. 2022, Varna: Science and Economics, 2022, 187-192., ISSN (online) 2815-3863 / https://www.ceeol.com/search/chapter-detail?id=1070060, individual.

2. „Planning of delivery routes using the linear programming model.“, The role of fundamental programs in university education: Proceedings from international scientific conference, organized by the department of Statistics and applied mathematics of the University of Economics – Varna, 21 oct. 2022, Varna: Science and Economics, 2022, 181-186., ISSN (online) 2815-3863 / https://www.ceeol.com/search/chapter-detail?id=1070057, individual.